Assignment 3

1. Generate spike trains from a Poisson model with a constant rate r of your choosing. To do this, divide time into sufficiently small intervals t and generate a spike within each interval with probability r t. From the spike trains you generated, compute:
   a) The distribution of interspike intervals.
   b) The coefficient of variation of the interspike intervals.
   c) The Fano factor for spike counts over a suitable time interval.
   Show that these match the analytic results obtained from the Poisson distribution.

2. Suppose you are given the spike times of n action potentials $t_1, t_2, t_3, \ldots t_n$ for a neuron responding to a stimulus $s(t)$. Assuming Poisson spiking, the log likelihood of these spikes over a time T, given a predicted response rate $r(t)$, is

   $$L = \sum_{i=1}^{N} \ln(r(t_i)) - \int_0^T dt\, r(t)$$

   A simple model of the firing rate is that it is proportional to $s(t)$, that is, $r(t) = ws(t)$, for some constant w. Find the value of w that maximizes the likelihood of the spike sequence.

3. In class, we derived the maximum likelihood estimate θ_{ML} for the parameter θ of a model in which the probability of observing x spikes in response to the presentation of a stimulus s is given by a linear-Poisson model:

   $$P(x) \sim \mathrm{Pois}(\lambda), \;\; \lambda = \theta s$$

   given a set of datapoints $\{x_i, s_i\}$. Use the same approach to derive an expression that θ_{ML} must satisfy for the linear-nonlinear-Poisson model:

   $$P(x) \sim \mathrm{Pois}(\lambda), \;\; \lambda = f(\theta s),$$

   where f is some known function. The solution will be an implicit expression for θ_{ML} involving f, f', x_i, and s_i.